THE OPEN UNIVERSITY OF SRI LANAKA

DIPLOMA IN TECHNOLOGY – LEVEL 03

FINAL EXAMINATION – 2010/2011

MPZ 3231 – ENGINERING MATHEMATICS IA

DURATION: THREE (03) HOURS

Date: 13th March 2011 **Time: 0930hrs – 1230hrs**

Instructions:

- **Answer only five (05) questions, selecting at least two(02) questions from each section A & B**
- **Number of pages in the paper – 05.**
- **Bold letters represent the vectors.**
- **All symbols are in standard notation.**

SECTION A

1. (a) Prove that the roots of the equation $|A-\lambda I|=0$ are -1 and 6 ,where I is the 2×2 unit matrix and $A=\begin{bmatrix}1 & 2\\ 5 & 4\end{bmatrix}$

Taking $\lambda_1=-1$ and $\lambda_2=6$, if $AX_1=\lambda_1 X_1$ and $AX_2=\lambda_2 X_2$ prove that $X_1=t\begin{bmatrix}1\\ -1\end{bmatrix}$ and $X_2=r\begin{bmatrix}2\\ 5\end{bmatrix}$,where t, r are parameters.

Prove that $\begin{bmatrix}1\\ -1\end{bmatrix}$ and $\begin{bmatrix}2\\ 5\end{bmatrix}$ are linearly independent.

If $U=\begin{bmatrix}1 & 2\\ -1 & 5\end{bmatrix}$ and $D=\begin{bmatrix}-1 & 0\\ 0 & 6\end{bmatrix}$, then show that $A=UDU^{-1}$.

Hence find A^n. Hint: You may use $\begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}^n=\begin{bmatrix}a^n & 0\\ 0 & b^n\end{bmatrix}$ where $n\in Z^+$.

Marks 150

(b) Find all the factors of $\begin{vmatrix}(b+c)^2 & a^2 & bc\\ (c+a)^2 & b^2 & ac\\ (a+b)^2 & c^2 & ab\end{vmatrix}$ by using row and column operations.

Marks 50

Please Turn Over.

2. (a) Prove that the normal line to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ is $y + tx = 2at + at^3$.

Prove that generally three normal lines can be drawn from a point P to the parabola $y^2 = 4ax$. If two of those normal lines are perpendicular to each other, then prove that the point P lies on the parabola $y^2 = a(x - 3a)$.

Marks 90

(b) (i) Find the coordinates of the point of intersection of the line $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z-4}{-1}$ and the plane $x + 2y - 2z = 9$.

(ii) Find the coordinates of the two points on the line $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z-4}{-1}$ such that the perpendicular distance from each of those two points on that line to the plane $x + 2y - 2z = 9$ is 8 units.

Marks 110

3. (a) If $ABCD$ is a regular tetrahedron and M is the mid point of CD, prove that the angle between BC and AM is $\cos^{-1}\left(\frac{\sqrt{3}}{6}\right)$.

Marks 40

(b) Define the vector product of two vectors.

(i) Vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are such that $\mathbf{a} \times \mathbf{b} = \mathbf{3a} \times \mathbf{c}$ .Prove that $\mathbf{a} = \mathbf{0}$ or $\mathbf{b} = \mathbf{3c} + k\mathbf{a}$ where $k \in R$ .If $|\mathbf{a}| = 5$, $|\mathbf{b}| = 3$, $|\mathbf{c}| = \sqrt{3}$ and $\mathbf{b} \cdot \mathbf{c} = 0$, then find the value of k.

For the above value of k find the angle between

(i) $\mathbf{a}$ and $\mathbf{b}$ (ii) $\mathbf{a}$ and $\mathbf{c}$.

(ii) $\overrightarrow{OA} = 2\mu\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{OB} = \mu\mathbf{i} + (\mu - 1)\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 4\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. Find the area of the triangle OAB and volume of the parallelepiped with the edges OA, OB, OC in terms of μ. Deduce the values of μ such that O, A, B, C are coplanar.

Marks 160

Please Turn Over

4. (a) If $x, y \in C$, prove that $|x-y| < |1-\bar{x}y|$ if and only if $\{ |x| < 1$ and $|y| < 1 \}$or

$\{ |x| > 1$ and $|y| > 1 \}$.

Marks 75

(b) State De Moivre's theorem .

If $z = \cos\theta + i\sin\theta$, then prove that $z^n + \dfrac{1}{z^n} = 2\cos n\theta$ and $z^n - \dfrac{1}{z^n} = 2i\sin n\theta$

Hence prove that $8\sin^4\theta = \cos 4\theta - 4\cos 2\theta + 3$ and $8\cos^4\theta = \cos 4\theta + 4\cos 2\theta + 3$

Marks 85

(c) Find the graph of $\mathrm{Re}(z+a) = |z-a|$, where $a > 0$.

Marks 40

Please Turn Over

SECTION B

5. (a) Prove that the general solution of the differential equation $\frac{dy}{dx} = \frac{x+3y}{3x+y}$ is

$(x+y) = \lambda(x-y)^2$. Here λ is an arbitrary constant.

Marks 80

(b) Show that the equation $2x(2x+3y^2)\frac{dy}{dx} + y(3x+2y^2) = 0$ is not exact. It is given that the above differential equation has an integrating factor of the form $x^h y^k$. Find the values of h and k such that the equation is exact. Hence find the general solution of the above differential equation.

Marks 120

6. (a) (i) Show that the real root of the equation $x^3 - 2 = 0$ lies between 1 and 2.

(ii) Prove that $x_{n+1} = \frac{2}{3}\left(x_n + \frac{1}{x_n^2} \right)$ is an iterative formula to evaluate the numerical value for $\sqrt[3]{2}$ (cubic root of 2). Deduce that x_n converges to $\sqrt[3]{2}$

(iii) Taking $x_1 = 1$ and using the above iterative formula, find the sixth iteration of $\sqrt[3]{2}$ .Hence obtain the numerical value of $\sqrt[3]{2}$ to two decimal places.

Marks 85

(b) Using the Jacobi's iteration method, find the sixth iteration of the solution of the following system of equations.

$$3x + 10y - z = -8$$
$$2x - 3y + 10z = 15$$
$$10x + y - 2z = 7$$

Marks 115

Please Turn Over

7. (a) Define the mean and variance of the continuous random variable. The probability density function of a continuous random variable X is

$$f(x) = \begin{cases} \lambda x(3-x)(3+x) & 0 \le x \le 3 \\ 0 & otherwise \end{cases}$$

i. Find the value of constant λ.

ii. Calculate the expectation, variance and mode of X

iii. Prove that the median of X is 1.62.

Marks 155

(b) State the Bayes theorem.

$U_1, U_2, U_3 \dots\dots\dots U_{13}$ are identical urns and contains 13 balls each which are only different in colour. U_r contains r number of red balls ,where $r = 1, 2, 3, \dots\dots 13$.

One ball is selected at random from an urn which is selected at random. If the colour of the selected ball is red ,find the probability that the selected urn is U_9.

Marks 45

8. (a) Given that the curve of $y = \dfrac{x^2 + x + 2}{x - 1}$ has an asymptote of the form. $y = mx + c$.

Find the equations of all the asymptotes, and also the maxima and minima of the above curve. Sketch the graph of the curve.

Marks 90

(b) (i) If $Z = \ln(x^2 + y^2)$, then prove that $\dfrac{\partial^2 Z}{\partial x^2} + \dfrac{\partial^2 Z}{\partial y^2} = 0$

(ii) If $u = \ln(x^2 + xy + y^2)$ and $y = e^x$, then find $\dfrac{du}{dx}$.

Marks 110

END

Copyright reserved